HOMEWORK 6
ECE 580
Due November 29, 2023

1. Design a Rauch filter with a dc gain = 1, $Q^2 = \frac{1}{2}$ and a pole frequency $f_0 = 20kHz$. Minimize the capacitance spread. Use $C_1 = 2pF$. Plot the gain vs. frequency response.

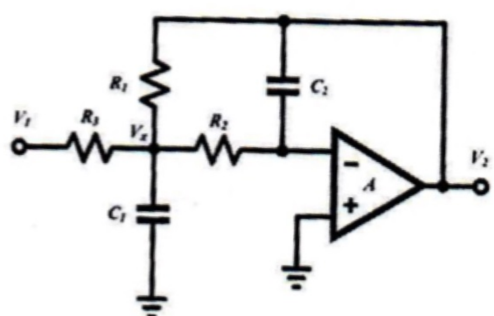
2. What is the -3 dB frequency of the Rauch filter with a given pole frequency and Q?

Solution: 1

To minimize the capacitance spread:

CMOS Active Filters, slide 15:

Write the denominator in terms of the pole frequency and pole Q. Use H(0). Equate the denominators. Express C2/C1 in terms of Q and G1/G2.Find the maximum of C2/C1.



- **Transfer function:**

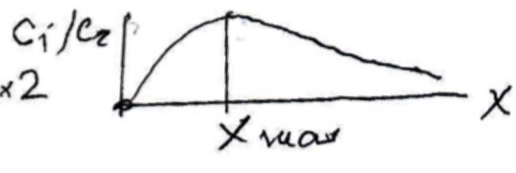
$$H(s) = \frac{G_2G_3/(C_1C_2)}{s^2 + s(G_1+G_2+G_3)/C_1 + G_1G_2/(C_1C_2) + \varepsilon}$$

$$s^2 + s\omega_0/Q \quad + \quad \omega_0^2$$

$H(0) = G_3/G_1 = 1 \mid \omega_0^2 = G_1 G_2 / C_1 C_2 \mid \omega_0/Q = (2G_1 + G_2)/C_1$

$$Q^2 = \omega_0^2 C_1^2 / (2G_1 + G_2)^2 = G_1 G_2 (C_1/C_2) / (2G_1 + G_2)^2$$

$y = C_2/C_1 = Q^{-2}\, x / (2x+1)^2 \,, \quad x \triangleq G_1/G_2$



$$\frac{dy}{dx} = Q^{-2}\,\frac{(2x+1)^2 - x(2x+1)\,2\times 2}{(\quad)^4}$$

$$4x_m^2 + \cancel{4x_m} + 1 - 8x_{max}^2 - \cancel{4x_{max}} = 0$$

$$4x_m^2 = 1 \quad x_{max} = \pm 1/2 \,, \quad G_1 = G_2/2$$

$$C_2/C_1 = Q^{-2}/8 \doteq 1/4 \,, \quad C_1 = 4C_2$$

2.

$$-\omega_3^2 + j\omega_3\omega_0/Q + \omega_0^2$$

$$(\omega_0^2 - \omega_3^2)^2 + \omega_0^2\omega_3^2/Q^2 = 2\omega_0^4$$

$$\cancel{\omega_0^4} - 2\omega_0^2\omega_3^2 + \omega_3^4 + \omega_0^2\omega_3^2/Q^2 = \cancel{2}\omega_0^4$$

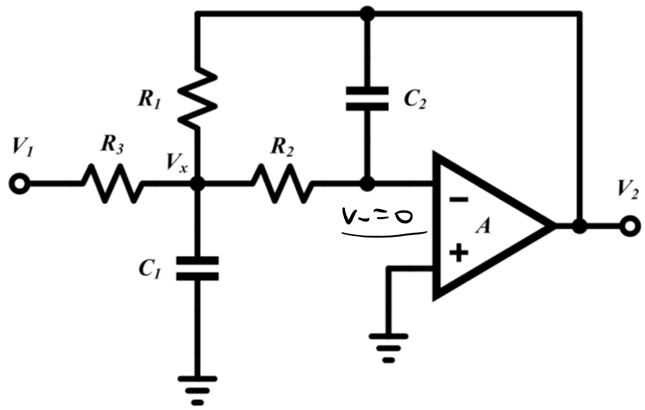
$$x \triangleq \left(\frac{\omega_3}{\omega_0}\right)^2 \;\Big|\; -2x + x^2 + x/Q^2 = 1$$

$$x^2 - (2 \cancel{- Q^{-2}})x - 1 = 0$$

$$x^2 = 1 \rightarrow \omega_3 = \omega_0$$

Solution

set 2.



1. Assume ideal opamp,

$$\begin{cases} @V_x \quad V_1(s)G_3 + V_2(s)G_1 - V_x(s)(G_1+G_2+G_3+sC_1) = 0 & ① \\ @\text{ A negative input} \quad V_x(s)G_2 + V_2(s)sC_2 = 0 & ② \end{cases}$$

From ②, $V_x(s) = -V_2(s)\frac{sC_2}{G_2}$ $\Rightarrow V_1(s)G_3 + V_2(s)G_1 + \frac{sC_2}{G_2}(G_1+G_2+G_3+sC_1)V_2(s) = 0$

$$\Rightarrow \frac{V_2(s)}{V_1(s)} = -\frac{G_3}{G_1 + \frac{sC_2}{G_2}(G_1+G_2+G_3+sC_1)} = -\frac{G_3G_2}{G_1G_2 + sC_2(G_1+G_2+G_3) + s^2C_1C_2}$$

$$= -\frac{G_2G_3/C_1C_2}{s^2 + s\frac{G_1+G_2+G_3}{C_1} + \frac{G_1G_2}{C_1C_2}}$$

DC gain: $\frac{G_3}{G_1} = 1 \Rightarrow G_1 = G_3$

Quality factor: $\frac{G_1+G_2+G_3}{C_1} = \frac{\omega_0}{Q}$ $\Rightarrow C_1 = \frac{Q}{\omega_0}(G_1+G_2+G_3) = \frac{Q}{\omega_0}(2G_1+G_2)$

pole frequency: $\frac{G_1G_2}{C_1C_2} = \omega_0^2$ $\Rightarrow C_2 = \frac{G_1G_2}{C_1\omega_0^2} = \frac{G_1G_2}{Q\omega_0(2G_1+G_2)}$

$$\Rightarrow \frac{C_1}{C_2} = \frac{Q^2(2G_1+G_2)^2}{G_1G_2} = Q^2\left(4\frac{G_1}{G_2} + \frac{G_2}{G_1} + 4\right) = \frac{1}{2}\left(4\frac{G_1}{G_2} + \frac{G_2}{G_1} + 4\right)$$

$\geq \frac{1}{2}\left(2\sqrt{4\frac{G_1}{G_2} * \frac{G_2}{G_1}} + 4\right) = 4$, [equal when $\frac{G_1}{G_2} = \frac{1}{2}$]. Therefore, C_1 is always larger than C_2.

Capacitance spread is minimized when $\frac{C_1}{C_2}$ minimized, i.e. $\frac{G_1}{G_2} = \frac{1}{2}$

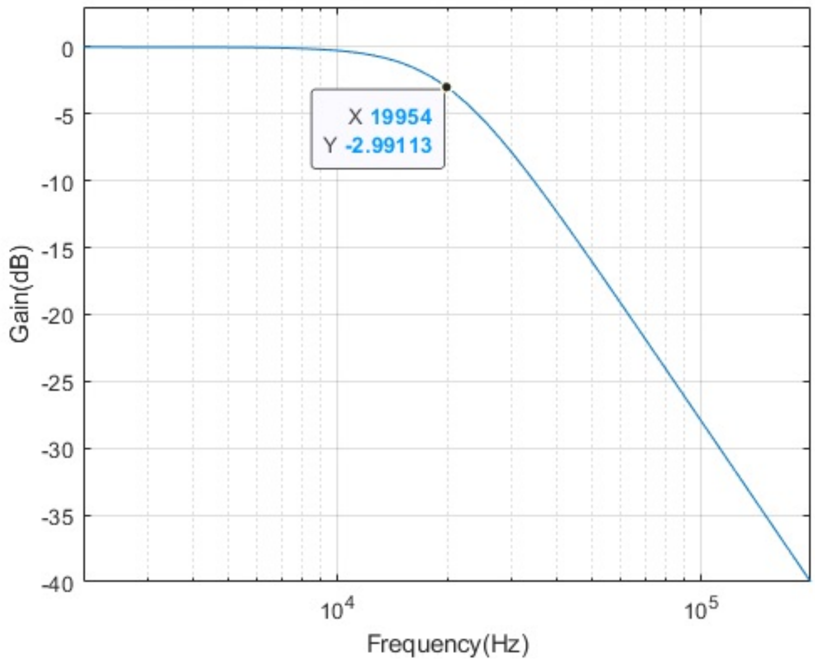
$G_2 = 2G_1$, $G_3 = G_1$ $\frac{C_1}{C_2} = 4$, $\Rightarrow C_2 = 0.5pF$

pole frequency ω_0. $\omega_0^2 = \frac{G_1G_2}{C_1C_2} \Rightarrow (2\pi \cdot 20\times10^3)^2 = \frac{G_1\, 2G_1}{2pF \cdot 0.5pF}$

$\Rightarrow G_1 = 20\sqrt{2}\pi$ nS ≈ 88.86 nS, $G_2 = 2G_1$ $G_3 = G_1$

$R_1 = 11.25M\Omega$ $R_2 = 5.63M\Omega$ $R_3 = 11.25M\Omega$ $C_1 = 2pF$ $C_2 = 0.5pF$

Gain vs. frequency:



2. $$H(s) = A\frac{\omega_0^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

$$|H(j\omega)| = \frac{1}{\sqrt{2}}|H_{DC}| \Rightarrow \left|A\frac{\omega_0^2}{(j\omega)^2+\frac{\omega_0}{Q}j\omega+\omega_0^2}\right| = \frac{1}{\sqrt{2}}A$$

$$\Rightarrow 2\omega_0^4 = (\omega_0^2-\omega^2)^2 + \left(\frac{\omega_0}{Q}\omega\right)^2$$

$$\Rightarrow \omega^4 + \left(\frac{1}{Q^2}-2\right)\omega_0^2\omega^2 - \omega_0^4 = 0$$

$$\Rightarrow \omega^2 = \frac{\left(2-\frac{1}{Q^2}\right)\omega_0^2+\sqrt{\left[\left(\frac{1}{Q^2}-2\right)\omega_0^2\right]^2+4\omega_0^4}}{2} = \frac{\left(2-\frac{1}{Q^2}\right)+\sqrt{\left(\frac{1}{Q^2}-2\right)^2+4}}{2}\omega_0^2$$

$$\Rightarrow \omega_{3dB} = \omega_0\left[\frac{\left(2-\frac{1}{Q^2}\right)+\sqrt{\left(\frac{1}{Q^2}-2\right)^2+4}}{2}\right]^{\frac{1}{2}} = \omega_0\sqrt{\left(1-\frac{1}{2Q^2}\right)+\sqrt{\frac{1}{4Q^4}-\frac{1}{Q^2}+2}}$$

Specially, when $Q^2=\frac{1}{2}$, $\omega_{3dB}=\omega_0$, as in part 1.